Physics Math Topics:

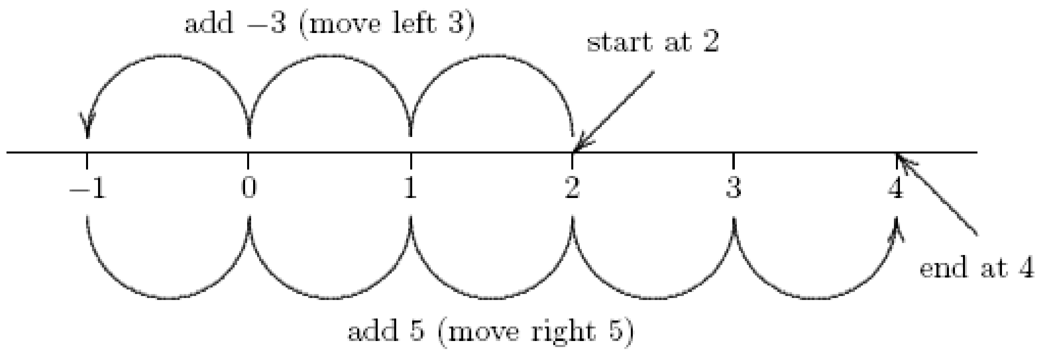
This is a list of the topics and methods in mathematics that you will be expected to do on a regular basis in your Physics class. At this level of your education, you should have addressed each of these in your math classes in the past.

You will find this list to be cumulative – if you do not understand an early concept, you will likely have problems with the next. This will extrapolate to you work in class.

Try to use each of these examples as a self-test. If you do not have a confident grasp of any of these concepts (that is, feel confident about **all** of the examples given below), is it important that you revisit them, or you will have serious issues in the course. Worksheets will be provided in class for extra practice.

These will be tested in your first quiz of the semester.

- **ARITHMETIC OF REAL NUMBERS:** Use of a number line



Basic Example:

4 - 2 = 2

Better Example:

-3 - 2 = -5

Best Example:

5 - (-3) = 8

- **SCIENTIFIC NOTATION:**

In science, we often end up with numbers that have so many digits that it is necessary to condense their expression into scientific notation. This entails splitting the number up into a mantissa (a number between one and ten) multiplied by an ordinate (ten raised to a certain power). The

Basic Example:

Express 4.4×10^6 in standard notation: 4,400,000
Express 0.0000000015 in scientific notation: 1.5×10^{-9}

Better Example:

Express 7/1000 in scientific notation: 7×10^{-3}

Best Example:

What is $(3x10^{8})x(1.5x10^{-5}) = 4.5x10^{3}$

- **ORDER OF OPERATIONS**:

When evaluating a multi-step arithmetic expression, it is customary to evaluate the operations in the following order:

- Parentheses
- Exponents (remember square roots, too!)
- Multiplication / Division
- Addition / Subtraction

If any of the operations are inside a parenthetical set, evaluate the inside according to the PEMDAS rules as well.

Basic Example:

$$4 + 2 \times 3 = 4 + (2 \times 3) = 4 + 6 = 10$$

Better Example:

$$\begin{aligned} 4 - 3[4 - 2(6 - 3)] \div 2 &= 4 - 3[4 - 2(3)] \div 2 \\ &= 4 - 3[4 - 6] \div 2 \\ &= 4 - 3[-2] \div 2 \\ &= 4 + 6 \div 2 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Best Example:

$$\begin{aligned} \frac{-9 + \sqrt{9^2 - 4 \times 2 \times 10}}{2 \times 2} &= \frac{(-9 + (9^2 - 4 \times 2 \times 10)^{1/2})}{(2 \times 2)} \\ &= \frac{(-9 + (81 - 4 \times 2 \times 10)^{1/2})}{(2 \times 2)} \\ &= \frac{(-9 + (81 - 80)^{1/2})}{(2 \times 2)} \\ &= \frac{(-9 + (1)^{1/2})}{(2 \times 2)} \\ &= \frac{(-9 + 1)}{(2 \times 2)} \\ &= \frac{(8)}{(4)} = 2 \end{aligned}$$

- **EVALUATING EQUATIONS:**

When given an equation composed of variables, such as a, b, c, x, y, z, etc, and values are given for all but one of them, the expression can be evaluated for that missing variable. This requires use of the skills noted above.

Basic Example:

$$y = mx + b$$
$$m = 2; b = 3; x = 4;$$

What is the value of *y* in this preceding expression?

$$\begin{aligned} y &= 2 \times 4 + 3 \\ &= 8 + 3 = 11 \end{aligned}$$

Better Example:

$$y = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$a = 2; b = 9; c = 10;$$

What is the value of *y* in this preceding expression?

$$= \frac{(-9 + (9^2 - 4 \times 2 \times 10)^{1/2})}{(2 \times 2)}$$
$$= \frac{(-9 + (81 - 4 \times 2 \times 10)^{1/2})}{(2 \times 2)}$$
$$= \frac{(-9 + (81 - 80)^{1/2})}{(2 \times 2)}$$
$$= \frac{(-9 + (1)^{1/2})}{(2 \times 2)}$$
$$= \frac{(-9 + 1)}{(2 \times 2)}$$
$$= \frac{(8)}{(4)} = 2$$

Best Example:

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$
$$a = 10; t = 5; v_0 = 0; x_0 = 15;$$

What is the value of *y* in this preceding expression?

$$x = \frac{1}{2}(10)(5)^2 + (0)(5) + 15$$
$$= \frac{1}{2}(10)(25) + 0 + 15$$
$$= 125 + 15 = 140$$

- **FOIL:**

To multiply polynomials, many use the foil method of distribution. Firsts, Outsides, Insides, Lasts.

Basic Example:

$$(x - 1)\ (x + 4) = x^2\ + 4x - x -\ 4 = x^2\ + 3x - 4$$

Better Example:

$$(x + 5)(x - 5) = x^2 + 5x - 5x - 25 = x^2 - 25$$

Best Example:

$$(-2x + 1)(3x + 2) = (-2 \cdot 3)x^2 + 3x + (-2 \cdot 2)x + 2$$
$$= -6x^2 + 3x - 4x + 2$$
$$= -6x^2 - x + 2$$

- **SOLVING FOR A VARIABLE:**

Isolating the variable to one side of the equation is a valuable skill and is necessary for success in any mathematics related discipline. To evaluate these expression, perform the reverse operations on the equation to get your variable of interest "alone."

Basic Example:

Solve for C.

$$A + B = C$$
$$\therefore A = C - B$$

Better Example:

Solve for t.

$$v = \frac{d}{t}$$
$$vt = d$$
$$t = \frac{d}{v}$$

Best Example:

Solve for x.

$$y = mx + b$$
$$y - b = mx$$
$$\frac{(y-b)}{m} = x$$

- **FACTORING:**
When faced with a multi order polynomial, the skills of factoring are often extremely important. While we will not address any complex factoring in this class, it is important to feel confident with these methods.

Basic Example:

$$x^2 - 3x = x(x-3)$$
$$x^2 + 5x - 6 = (x-1)(x+3)$$

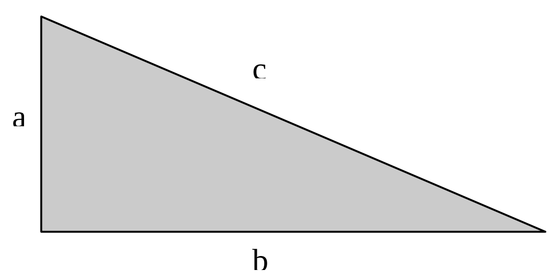
Better Example:

$$x^2 - 25 = (x-5)(x+5)$$

Best Example:

$$6x^2 - 13x - 5 = (2x-5)(3x+1)$$

- **PYTHAGOREAN THEOREM:** In a right triangle, the lengths of the sides of the triangle evaluate to this expression, where *a* and *b* are the lengths of the two sides straddling the right angle and *c* is the length of the hypotenuse. (Notice that, in the case of a right triangle, the longest side is always the hypotenuse.)



$$a^2 + b^2 = c^2$$

There are several right triangles whose sides are integers. There are 3-4-5, 6-8-10, 5-12-13. Memorizing these will help you become more efficient with these problems.

Basic Example:

If you know that the sides of a triangle sides are of lengths 3 and 4, what is the length of the hypotenuse? (5)

Better Example:

If a ladder is 3 meters long and the foot rests 1 meter from the wall, how high up is it? $\sqrt{2}$

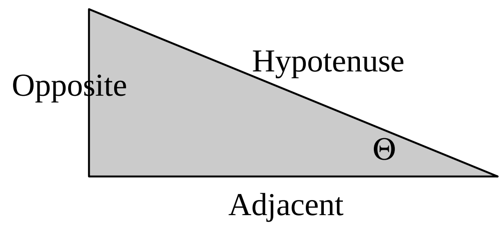
Best Example:

If the hypotenuse has a length of 1, and one side has a length of 0.5, what is the length of the third side? $\frac{\sqrt{3}}{2}$

If the hypotenuse has a length of 1, and the triangle is an isosceles triangle, what is the length of the other two sides? $\frac{\sqrt{2}}{2}$

- SOHCAHTOA

In trigonometric applications, information can be gleaned about a right triangle based on its sides and angles. This is important because it is a method that allows us to determine the angles of a right triangle based on the lengths of its sides, and vice versa.



The most common way to remember the relationships is using SOH-CAH-TOA. That is:

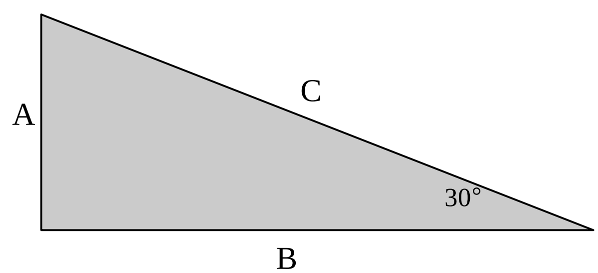
$$\sin\Theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\Theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\Theta = \frac{Opposite}{Adjacent}$$

Basic Example:

Which side is… adjacent to the angle? (B)
… opposite to the angle? (A)
…the hypotenuse? (C)



Better Example:

Which trigonometric function is equivalent to $\frac{A}{B}$? $\frac{A}{B} = \frac{Opposite}{Adjacent} = \tan 30°$

Best Example:

If A has a length of 5, what is the length of C?

$$\sin\Theta = \frac{Opposite}{Hypotenuse}$$

$$\sin 30° = \frac{A}{C} = \frac{5}{C}$$

$$C = 5\sin 30° = 5 \cdot \frac{1}{2} = 2.5$$

- **COMMON TRIGONOMETRIC FUNCTIONS**

In the Trigonometric Unit Circle, Sin corresponds to the *y* value or height of the new right triangle. Cos corresponds to the *x* value or width of the new right triangle. Tan is the ratio:

$$\tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{y}{x} = slope$$

Θ	sin Θ	cos Θ	tan Θ
0	0	1	0
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	Undef

Know the values in this chart, either by using a deep understanding of the unit circle to derive each number, or just by memorizing it.